

Two- and Three-Pion Interferometry for a Nonchaotic Source in Relativistic Nuclear Collisions

Hiroki Nakamura⁽¹⁾ and Ryoichi Seki^(2,3)

⁽¹⁾ *Department of Physics, Waseda University, Tokyo 169-8555, Japan*

⁽²⁾ *Department of Physics, California State University, Northridge, CA 91330*

⁽³⁾ *W. K. Kellogg Radiation Laboratory, 106-38, California Institute of Technology,
Pasadena, CA 91125*

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Abstract

Two- and three-pion correlation functions are investigated for a source that is not fully chaotic. Various models are examined to describe the source. The chaoticity and weight factor are evaluated in each model as measures of the strength of correlations and compared to experimental results. A new measure of three-pion correlation is also suggested.

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I. INTRODUCTION

Two-pion correlations obtained in relativistic heavy-ion collisions have been used to extract the size and shape of the pion-emitting source, based on the Hanbury-Brown Twiss (HBT) effect. Correlations are not, of course, limited to those of two pions, but can also be of multi-pions. Though multi-pion correlations are unavoidably complicated, they contain new information not available from the two-pion correlations. For example, when final-state interactions of the emitted pions are neglected, the two-pion correlations for a chaotic source depend only on the magnitude of the Fourier transform of the pion-source function, but the multi-pion correlations depend also on its phase [1,2]. The magnitude is an even function of the relative momentum of the emitted pions, while the phase is an odd function. One thus hopes to extract new information about the source from the multi-pion correlations.

In a previous article [2], we made a detailed investigation of three-pion correlations, the simplest multi-pion correlations after the two-pion correlations, over a wide range of kinematics, to investigate the feasibility of extracting new information about the source through the phase in the case of a chaotic source. We find that extracting new information would be rather difficult in practice because the multiplicative factor of the phase function becomes small in the region where the phase varies appreciably.

In experiments, the two-pion correlations at the zero relative momentum is observed to be less than two. After the final-state interactions are removed, it must be two in the case of a chaotic source. The measurement suggests that the source is not completely chaotic. Recently, a measurement of the three-pion correlations has been reported, showing that the strength of the three-pion correlations are also less than expected in the case of a chaotic source.

In this paper we investigate the two- and three-pion correlations for a source not completely chaotic. We introduce models of the source that are various mixtures of coherent and chaotic sources. The models also include one with a novel structure, a mixture of multiple coherent sources and a chaotic source. These models describe different dynamics generating pion emissions, though we do not pursue the identification of dynamics corresponding to each model.

A mixture of a coherent source and a chaotic source is a model that has often appeared in the literature [3,4,1] under the name “partially coherent source.” We find that this popular model poorly reproduces the recent data of the two- and three-pion correlations [5]. A model consisting of multi-coherent sources and one chaotic source appears to yield a good agreement.

It is both theoretical and experimental practice to calculate the so-called weight factor from the three-pion correlations at the vanishing relative momenta. The weight factor is usually considered to represent the strength of the genuine three-pion correlations. When we examine more complicated models than the popular partially coherent model, we find that the weight factor no longer describes the genuine strength. The expression that yields the genuine strength depends on the structure of the source, and there is no universal expression as such. Nevertheless, we propose a modified expression for the weight factor that has a wider validity.

In Sec. II, we define pion spectra, correlation functions, and measures of two- and three-pion correlation functions, chaoticity, and weight factor, respectively. In Sec. III,

various nonchaotic models of the pion-emitting source are introduced, and chaoticities and weight factors are obtained for them. Section IV presents discussions and a summary: a new expression of the weight factor is introduced and discussed, and chaoticities and weight factors are compared with the recent experimental data. Appendix A gives a derivation of correlations for multi-coherent sources. In Appendix B, the new weight factor is derived.

II. CORRELATION FUNCTIONS, CHAOTICITY, AND WEIGHT FACTOR

For the sake of clarity in the sections to follow, we define correlation functions and their measures, chaoticity, and weight factor. Note that our definition is standard.

We first write the basic quantities, pion spectra, in second-quantization form, as follows:

$$W_1(p) = \langle a_p^\dagger a_p \rangle, \quad (1)$$

$$W_2(p_1, p_2) = \langle a_{p_1}^\dagger a_{p_2}^\dagger a_{p_1} a_{p_2} \rangle, \quad (2)$$

$$W_3(p_1, p_2, p_3) = \langle a_{p_1}^\dagger a_{p_2}^\dagger a_{p_3}^\dagger a_{p_1} a_{p_2} a_{p_3} \rangle. \quad (3)$$

Though these expressions are simple and reasonable, the explicit definition of $\langle \dots \rangle$ is a complicated issue. $\langle \dots \rangle$ represents a quantum statistical average and is formally written as $\langle \psi | \dots | \psi \rangle$ in terms of the quantum state, $|\psi\rangle$, or $\text{Tr}\{\hat{\rho} \dots\}$ in terms of the density matrix, $\hat{\rho}$. In this work, we introduce models to represent various underlying dynamics of the pion emission. Note that the momenta above and hereafter are on-shell, e.g., $p^0 = \sqrt{\mathbf{p}^2 + m^2}$ for p .

In terms of the spectra, we define two- and three-pion correlation functions in the usual way [3],

$$C_2(p_1, p_2) = \frac{\langle n \rangle^2}{\langle n(n-1) \rangle} \frac{W_2(p_1, p_2)}{W_1(p_1)W_1(p_2)}, \quad (4)$$

$$C_3(p_1, p_2, p_3) = \frac{\langle n \rangle^3}{\langle n(n-1)(n-2) \rangle} \frac{W_3(p_1, p_2, p_3)}{W_1(p_1)W_1(p_2)W_1(p_3)}. \quad (5)$$

Here, the normalizations are introduced in order to take account of multiplicity fluctuation, with the following definitions:

$$\langle n \rangle = \int d^3p W_1(p), \quad (6)$$

$$\langle n(n-1) \rangle = \int d^3p_1 d^3p_2 W_2(p_1, p_2), \quad (7)$$

$$\langle n(n-1)(n-2) \rangle = \int d^3p_1 d^3p_2 d^3p_3 W_3(p_1, p_2, p_3). \quad (8)$$

In the usual models, the above normalizations yield C_2 and $C_3 \rightarrow 1$ as the relative momenta approach infinity. In more complicated models, such as those that possess the particle-number fluctuation per mode, however, the normalizations of Eqs. (4) and (5) yield an asymptotic value different from unity. We will discuss this point fully in Sec. IIID.

When the HBT effect does not appear, correlation functions are independent of relative momenta. When the HBT effect occurs, however, the correlation functions are no longer

constant, but are dependent on relative momenta. As is well known, the size and shape of a pion-emitting source are extracted from the relative-momentum dependence of the correlations through the HBT effect. The methods of extracting the size and shape of the source has been discussed numerous times, and we will not go into the issues here.

We define the chaoticity, $\lambda(p)$, and the weight factor, $\omega(p)$, which are usually considered to represent measures of the strength of the two- and three-pion correlations, respectively [1,5]:

$$\lambda(p) = C_2(p, p) - 1, \quad (9)$$

$$\omega(p) = \frac{C_3(p_1, p_2, p_2) - 1 - (C_2(p_1, p_2) - 1) - (C_2(p_2, p_3) - 1) - (C_2(p_3, p_1) - 1)}{2\sqrt{(C_2(p_1, p_2) - 1)(C_2(p_2, p_3) - 1)(C_2(p_3, p_1) - 1)}} \Big|_{p_1=p_2=p_3=p}. \quad (10)$$

Note that we define the weight factor explicitly at $p_1 = p_2 = p_3$, but that it has been extracted from measurements over small relative momenta [5]. The two methods show practically no difference since the weight factor is expected to be a slowly varying function of the relative momenta [1,2].

The weight factor, however, is not always the measure of the strength of the genuine three-pion correlations. Generally, the three-pion correlation functions are related to the two-pion correlation functions as

$$\begin{aligned} C_3(p_1, p_2, p_3) = & 1 + a(C_2(p_1, p_2) + C_2(p_2, p_3) + C_2(p_3, p_1) - 3) \\ & + (\text{genuine three-pion correlation}) \\ & + (\text{other two-pion correlation}), \end{aligned} \quad (11)$$

where the coefficient, a , is not always unity even at $p_1 = p_2 = p_3$. Consequently, Eq. (10) does not always remove the linear dependence of C_2 's from C_3 . In the cases of a chaotic source and a partially coherent source, a is unity, but for sources of more complicated structure, it is not. Furthermore, C_3 generally depends on the two-pion correlations not only as the linear C_2 's but also in more complicated ways as a function of two momenta. We will discuss these points fully in the following section.

The relation between C_3 and C_2 depends on dynamics and is model-dependent in practice. The extraction of the strength of the genuine three-pion correlations thus requires explicit knowledge of the dynamics. Accordingly, in this paper we adopt the conventional approach of using the weight factor defined as Eq. (10), for numerical results as a convenient means for making comparisons among various models.

III. DIFFERENT MODELS OF THE SOURCE

A. Coherent Source and Chaotic Source

The spectra are often written by c -number source current [3,6], yielding a chaotic source and a coherent source as the two extreme cases. The chaotic source shows the HBT effect

with both the chaoticity and the weight factor as unity. The correlation functions for a coherent source are independent of the relative momenta, and the chaoticity for it is zero.

The difference between the out-state and in-state annihilation operators of an emitted pion defines the source current as

$$\begin{aligned} a_{\text{out}}(\mathbf{p}) &= a_{\text{in}}(\mathbf{p}) + i \int d^4x \frac{1}{\sqrt{(2\pi)^3 \cdot 2p^0}} J(x) e^{-ip \cdot x} \\ &= a_{\text{in}}(\mathbf{p}) + i \frac{1}{\sqrt{(2\pi)^3 \cdot 2p^0}} J(p), \end{aligned} \quad (12)$$

where $J(x)$ and $J(p)$ are the source current and its Fourier transform, respectively. The average that appears in Eqs. (1)-(3) can be written as

$$\langle \cdots \rangle = \int \mathcal{D}J^*(p) \mathcal{D}J(p) \mathcal{P}[J^*(p), J(p)] \langle 0_{\text{in}} | \cdots | 0_{\text{in}} \rangle, \quad (13)$$

where $|0_{\text{in}}\rangle$ is the in-state vacuum and $\mathcal{P}[J^*(p), J(p)]$ is the distribution functional of $J(p)$, which has a statistical fluctuation. The pion spectra are obtained in this case from Eqs. (1)-(3):

$$W_1(p) = \frac{1}{(2\pi)^3 \cdot 2p^0} \langle |J(p)|^2 \rangle_J, \quad (14)$$

$$W_2(p_1, p_2) = \frac{1}{(2\pi)^6 \cdot 2p_1^0 \cdot 2p_2^0} \langle |J(p_1)|^2 |J(p_2)|^2 \rangle_J, \quad (15)$$

$$W_3(p_1, p_2, p_3) = \frac{1}{(2\pi)^9 \cdot 2p_1^0 \cdot 2p_2^0 \cdot 2p_3^0} \langle |J(p_1)|^2 |J(p_2)|^2 |J(p_3)|^2 \rangle_J, \quad (16)$$

where $\langle \cdots \rangle_J = \int \mathcal{D}J^*(p) \mathcal{D}J(p) \mathcal{P}[J^*(p), J(p)] \cdots$. Hereafter, we do not explicitly show the subscript J of the angle bracket. If the phase of the source current is random, we call the source *chaotic*. In this case, $\mathcal{P}[J^*(p), J(p)]$ have a Gaussian form [6] and higher-order moments of $J(p)$ are represented by the second-order moment such as

$$\begin{aligned} \langle J^*(p_1) J^*(p_2) J(q_1) J(q_2) \rangle &= \\ \langle J^*(p_1) J(q_1) \rangle \langle J^*(p_2) J(q_2) \rangle &+ \langle J^*(p_1) J(q_2) \rangle \langle J^*(p_2) J(q_1) \rangle. \end{aligned} \quad (17)$$

For a chaotic source, the pion spectra are written as

$$W_1(p_1) = F_{11}, \quad (18)$$

$$W_2(p_1, p_2) = F_{11} F_{22} + F_{12}^2, \quad (19)$$

$$W_3(p_1, p_2, p_3) = F_{11} F_{22} F_{33} + \sum_{(a,b,c)} F_{aa} F_{bc}^2 + 2F_{12} F_{23} F_{31} \cos(\Phi_{12} + \Phi_{23} + \Phi_{31}), \quad (20)$$

where the amplitude, F_{ab} , and the phase, Φ_{ab} are defined as [1]

$$F_{ab} \exp(i\Phi_{ab}) \equiv \frac{1}{(2\pi)^3 \sqrt{2p_a^0 \cdot 2p_b^0}} \langle J^*(p_a) J(p_b) \rangle. \quad (21)$$

$\sum_{(a,b,c)}$ implies a sum over $(a, b, c) = (1, 2, 3), (2, 3, 1), (3, 1, 2)$. The correlation functions are then

$$C_2(p_1, p_2) = 1 + \frac{F_{12}^2}{F_{11}F_{22}}, \quad (22)$$

$$C_3(p_1, p_2, p_3) = 1 + \sum_{(a,b)} \frac{F_{ab}^2}{F_{aa}F_{bb}} + 2 \frac{F_{12}F_{23}F_{31}}{F_{11}F_{22}F_{33}} \cos(\Phi_{12} + \Phi_{23} + \Phi_{31}), \quad (23)$$

where $\sum_{(a,b)}$ is over $(a, b) = (1, 2), (2, 3), (3, 1)$. F_{ab} and Φ_{ab} contain the information about the size and shape of the pion-emitting source, due to the HBT effect. The chaoticity and weight factor for a chaotic source always achieve unity.

When the source current has no randomness, the source is called *coherent*. In this case, the n -pion spectra are expressed as

$$W_n(p_1, \dots, p_n) = \prod_{l=1}^n W_1(p_l). \quad (24)$$

The HBT effect does not appear in this case. The correlation functions achieve unity, and the chaoticity vanishes.

B. Partially Coherent Source

In nuclear collisions, one may expect to involve the mixture of a coherent source and a chaotic source, as suggested by the observation that the chaoticities are often smaller than unity. Here, we sketch the case of the partially coherent source, mostly following Ref. [1].

When the pion-emitting source is a mixture of a chaotic source and a coherent source, the source current is written as a sum of both currents, $J(x) = J_{cha}(x) + J_{coh}(x)$ [1,3]. The sources are not correlated with each other, $\langle J_{cha}^*(p)J_{coh}(q) \rangle = 0$. The one-pion spectrum and correlation functions for this source are obtained as

$$W_1(p_1) = f_{11} + F_{11}, \quad (25)$$

$$C_2(p_1, p_2) = 1 + \frac{F_{12}^2 + 2f_{12}F_{12} \cos(\Phi_{12} - \phi_{12})}{(f_{11} + F_{11})(f_{22} + F_{22})}, \quad (26)$$

$$\begin{aligned} C_3(p_1, p_2, p_3) = 1 + \sum_{(a,b)} \frac{F_{ab}^2 + 2f_{ab}F_{ab} \cos(\Phi_{ab} - \phi_{ab})}{(f_{aa} + F_{aa})(f_{bb} + F_{bb})} \\ + 2 \frac{1}{(f_{11} + F_{11})(f_{22} + F_{22})(f_{33} + F_{33})} \left\{ F_{12}F_{23}F_{31} \cos(\Phi_{12} + \Phi_{23} + \Phi_{31}) \right. \\ \left. + \sum_{(a,b,c)} [f_{ab}F_{bc}F_{ca} \cos(\phi_{ab} + \Phi_{bc} + \Phi_{ca})] \right\}, \end{aligned} \quad (27)$$

where

$$f_{ab} \exp(i\phi_{ab}) \equiv \frac{1}{(2\pi)^3 \sqrt{2p_a^0 \cdot 2p_b^0}} J_{coh}^*(p_a) J_{coh}(p_b), \quad (28)$$

and F_{ab} and Φ_{ab} are the same as those for a chaotic source, Eq. (21). The chaoticity and weight factor are evaluated as

$$\lambda(p) = \epsilon(p)(2 - \epsilon(p)), \quad (29)$$

$$\omega(p) = \sqrt{\epsilon(p)} \frac{3 - 2\epsilon(p)}{(2 - \epsilon(p))^{3/2}}, \quad (30)$$

where $\epsilon(p)$ is the fractional parameter of the coherent source:

$$\epsilon(p_1) = \frac{F_{11}}{f_{11} + F_{11}}. \quad (31)$$

The source becomes chaotic for $\epsilon(p) = 1$, while it becomes coherent for $\epsilon(p) = 0$. Figure 1 shows the chaoticity and weight factor as functions of $\epsilon(p)$, and in Fig. 5 the weight factor is shown as a function of chaoticity, varying as $\epsilon(p)$. The weight factor can vary from 0 to 1. In this case, the term -3λ , in Eq. (10), completely removes the two-pion correlations.

C. Multi-Coherent Sources

We consider the mixture of a small number of coherent sources. This model differs from the case of multiple coherent sources that was previously examined in [3] and, to the best of our knowledge, this is the first time that this model has been explicitly discussed. We assume here that the pion-emitting source is made of N coherent sources that are not coherent with each other and *appear to be obeying the Poisson distribution*. Note that the model considered in [3] is a mixture of multiple coherent sources that are randomly distributed, and that the model was introduced as a description of a chaotic source when the number of the coherent sources becomes large. In a sense our model here is the opposite limit of the small number of coherent sources in the model considered in [3]. In the following, we present merely the final expressions, leaving their derivations for Appendix A.

The source current can be written as

$$J(x) = \sum_{n=1}^N j(x - X_n) e^{-i\theta_n}, \quad (32)$$

where $j(x - X_n)$ is the n -th coherent source current, located at X_n with the random phase θ_n . Each coherent source is assumed to be expressed by the same $j(x)$ but to be located at a different position. There are N coherent sources, obeying the Poisson distribution. Note that the distribution must be renormalized in order to exclude the no-source event that is not observed.

The one-pion spectrum and normalized correlation functions are obtained as

$$W(p_1) = a_1 \frac{\alpha}{1 - e^{-\alpha}}, \quad (33)$$

$$C_2(p_1, p_2) = 1 + \frac{\alpha}{\alpha + 1} |\rho_{12}|^2, \quad (34)$$

$$C_3(p_1, p_2, p_3) = 1 + \frac{\alpha(\alpha + 2)}{\alpha^2 + 3\alpha + 1} \sum_{(a,b)} |\rho_{ab}|^2 + \frac{\alpha^2}{\alpha^2 + 3\alpha + 1} \cdot 2\text{Re}(\rho_{12}\rho_{23}\rho_{31}), \quad (35)$$

where ρ_{ij} is the Fourier transform of the spatial distribution of coherent sources, and α is a parameter of the Poisson distribution. The mean number of coherent sources is $\alpha/(1 - \exp(-\alpha))$. In this case, the subtraction -3λ , in Eq. (10), does not remove the two-pion correlations completely, because the coefficient of $|\rho_{12}|^2$ in Eq. (34) is different from that in Eq. (35). The chaoticity and weight factor are derived as

$$\lambda = \frac{\alpha}{\alpha + 1}, \quad (36)$$

$$\omega = \frac{1}{2} \frac{2\alpha^2 + 2\alpha + 3}{\alpha^2 + 3\alpha + 1} \sqrt{\frac{\alpha + 1}{\alpha}}. \quad (37)$$

In Fig. 2, the chaoticity and weight factor are illustrated as functions of the mean number of sources, and Fig. 5 shows the weight factor as a function of the chaoticity. The chaoticity varies from 0 to 1, because this source becomes coherent at $\alpha = 0$ and chaotic at $\alpha \rightarrow \infty$. The weight factor diverges at $\alpha \rightarrow 0$. The reason for this is the failure of the subtraction -3λ , in Eq. (10), and the divergence is not caused by the genuine three-pion correlation.

D. Multi-Coherent Sources and One Chaotic Source

We now consider the source in which the multi-coherent sources of the previous subsection are mixed with a chaotic source. The multi-coherent sources are not coherent with each other, as before. The source current is written as

$$J(x) = \sum_{n=1}^N j(x - X_n) e^{-i\theta_n} + J_{cha}(x), \quad (38)$$

where $j(x)$ and $J_{cha}(x)$ are a coherent source current and the chaotic source current, respectively. The n -th coherent source is located at X_n and is distributed with $\rho(X_n)$. The phase, θ_n , is randomly distributed between 0 and 2π . There are N coherent sources, obeying the Poisson distribution, Eq. (A9).

Pion spectra come out to be

$$W_1(p_1) = \alpha h_{11} + F_{11}, \quad (39)$$

$$W_2(p_1, p_2) = W_1(p_1)W_1(p_2) + \alpha h_{11}h_{22} + \left| \alpha h_{12}e^{i\psi_{12}} + F_{12}e^{i\Phi_{12}} \right|^2, \quad (40)$$

$$\begin{aligned} W_3(p_1, p_2, p_3) = & W_1(p_1)W_1(p_2)W_1(p_3) + (3\alpha + 1)\alpha h_{11}h_{22}h_{33} + \alpha \sum_{(a,b,c)} h_{aa}h_{bb}F_{cc} \\ & + \sum_{(a,b,c)} W_1(p_a) \left| \alpha h_{bc}e^{i\psi_{bc}} + F_{bc}e^{i\Phi_{bc}} \right|^2 \\ & + 2 \sum_{(a,b,c)} h_{aa} \left(\alpha^2 |h_{bc}|^2 + \alpha h_{bc}F_{bc} \cos(\Phi_{bc} - \psi_{bc}) \right) \\ & + 2\text{Re} \left\{ (\alpha h_{12}e^{i\psi_{12}} + F_{12}e^{i\Phi_{12}})(\alpha h_{23}e^{i\psi_{23}} + F_{23}e^{i\Phi_{23}})(\alpha h_{31}e^{i\psi_{31}} + F_{31}e^{i\Phi_{31}}) \right\}, \end{aligned} \quad (41)$$

where α is the parameter in the Poisson distribution, and

$$h_{ij}e^{i\psi_{ij}} = \frac{j^*(p_i)j(p_j)}{(2\pi)^3\sqrt{2p_1^0 \cdot 2p_2^0}}\rho_{ij}. \quad (42)$$

For $|p_1 - p_2| \rightarrow \infty$, we have

$$\frac{W_2(p_1, p_2)}{W_1(p_1)W_1(p_2)} \rightarrow 1 + \frac{1}{\alpha}(1 - \epsilon(p_1))(1 - \epsilon(p_2)), \quad (43)$$

where the fractional parameter of the chaotic source, ϵ , is defined as

$$\epsilon(p_1) = \frac{F_{11}}{\alpha h_{11} + F_{11}}. \quad (44)$$

The normalization factor of $C_2(p_1, p_2)$, shown in Eq. (4), does not yield the proper asymptotic value of unity as the relative momentum approaches infinity. Generally ϵ depends on the momentum, and the asymptotic value of C_2 thus depends on the two momenta separately. If we assume that ϵ is independent of the momentum, we find that C_2 is normalized properly by the use of Eqs. (4) and (5). In this case, the correlation functions are given by

$$\begin{aligned} C_2(p_1, p_2) &= 1 + \frac{|\alpha h_{12}e^{i\psi_{12}} + F_{12}e^{i\Phi_{12}}|^2}{W_1(p_1)W_1(p_2) \left(1 + \frac{1}{\alpha}(1 - \epsilon)^2\right)}, \quad (45) \\ C_3(p_1, p_2, p_3) &= 1 + \frac{1}{1 + \frac{3\alpha+1}{\alpha^2}(1 - \epsilon)^3 + \frac{3}{\alpha}(1 - \epsilon)^2\epsilon} \left[\sum_{(a,b)} \frac{|\alpha h_{ab}e^{i\psi_{ab}} + F_{ab}e^{i\Phi_{ab}}|^2}{W_1(p_a)W_1(p_b)} \right. \\ &\quad + \frac{2\text{Re} \left\{ (\alpha h_{12}e^{i\psi_{12}} + F_{12}e^{i\Phi_{12}})(\alpha h_{23}e^{i\psi_{23}} + F_{23}e^{i\Phi_{23}})(\alpha h_{31}e^{i\psi_{31}} + F_{31}e^{i\Phi_{31}}) \right\}}{W_1(p_1)W_1(p_2)W_1(p_3)} \\ &\quad \left. + 2 \sum_{(a,b)} \left\{ \frac{\alpha(1 - \epsilon)h_{ab}^2}{W_1(p_a)W_1(p_b)} + \frac{(1 - \epsilon)h_{ab}F_{ab} \cos(\Phi_{ab} - \psi_{ab})}{W_1(p_a)W_1(p_b)} \right\} \right]. \quad (46) \end{aligned}$$

Comparison of the preceding C_2 and the first term in the square bracket in C_3 shows that the two-pion correlations are not to be removed completely, as $a \neq 1$ in Eq. (11) in this case. The conventional weight factor thus no longer represents the strength of the genuine three-pion correlations. Furthermore, the last sum in the square bracket of the above C_3 represents the effects of the two-pion correlation (because it depends on a pair of the momenta), corresponding to the “other two-pion correlation” in Eq. (11).

The chaoticity and weight factor are obtained as

$$\lambda = \frac{\alpha}{\alpha + (1 - \epsilon)^2}, \quad (47)$$

$$\omega = \frac{2\alpha^2 + 2\alpha(1 - \epsilon)^2 + 3(1 - \epsilon)^3(1 - 2\epsilon)}{2(\alpha^2 + 3\alpha(1 - \epsilon)^2 + (1 - \epsilon)^3)} \sqrt{\frac{\alpha + (1 - \epsilon)^2}{\alpha}}. \quad (48)$$

Figures 3 and 4 show λ and ω , respectively, as functions of α at $\epsilon = 0.1 - 0.9$. In Fig. 6, the weight factor is shown as a function of the chaoticity, varying ϵ for various α 's. The divergence of ω at $\alpha = 0$, except for $\epsilon = 0.5$, is caused by the incomplete cancellation of the C_2 's.

IV. DISCUSSIONS AND SUMMARY

We have examined various models of the source that are not completely chaotic. The chaoticity comes out to be between 0 and 1 in all models, but the weight factor takes a wide range of the value. The value of the weight factor even diverges in some cases, as a consequence of incomplete removal of the two-pion correlations from C_3 . For a partially coherent source that is commonly examined, the removal is complete and the anomalous behavior does not appear.

There is a way to avoid such incomplete removal, at least in all the models that we have examined here. It is done through a new subtraction,

$$R_3(p_1, p_2, p_3) = C_3(p_1, p_2, p_3) - 1 - \frac{\langle n \rangle (2\langle n(n-1) \rangle - \langle n \rangle^2)}{\langle n(n-1)(n-2) \rangle} \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} \sum_{(a,b)} (C_2(p_a, p_b) - 1), \quad (49)$$

where $\langle n \rangle$ and the similar are defined in Eqs. (6) - (8). This subtraction works correctly for the multi-coherent sources and also for the mixture of multi-coherent sources and one chaotic source (but at the zero relative momenta). We find that the terms corresponding to the “other two-pion correlations” also vanish at the zero relative momenta (see Appendix B). Using Eq. (49), we define a new weight factor as

$$\omega' = \frac{\langle n(n-1)(n-2) \rangle}{\langle n \rangle^3} \frac{1}{2} R_3(p_1, p_2, p_3) \left(\frac{\langle n(n-1) \rangle}{\langle n \rangle^2} \sum_{(a,b)} \{C_2(p_a, p_b) - 1\} \right)^{-\frac{3}{2}}. \quad (50)$$

For the partially coherent source, the new weight factor becomes the conventional weight factor of Eq. (10).

Though the new weight factor properly represents the strength of the genuine three-pion correlations for all models examined in this work, the new weight factor must not have this property at all times. The relation between C_2 's and C_3 is generally complicated, depending on dynamics in each case. We thus do not expect that the new weight factor has a universal application.

Figures 5 and 6 illustrate the weight factors as functions of the chaoticities for the different models, together with the experimental data from the CERN NA44 Collaboration [5]. In this experiment, the chaoticity and weight factor are measured as $0.4 - 0.5$ and 0.20 ± 0.19 , respectively. Figure 5 shows that the partially coherent source and the multi-coherent sources disagree with the experiment. The mixture of multi-coherent sources and one chaotic source reproduces the data if we set $\alpha = 0.13$ and $\epsilon = 0.60$. This corresponds to the mean number of the coherent sources being 0.13 and to about 60 % of the total pions emitted from the chaotic source. The ratio of the pion number emitted from one coherent source to that from one chaotic source, h_{11}/F_{11} , is then about 5. This ratio may be unrealistically large, but we note that the experimental data are “minimum-bias,” suggesting that our best-fit may not be unrealistic since the multiplicity fluctuation can be large. Further data are needed to confirm that this is indeed the case.

In summary, we investigate the two- and three-pion correlations for various models of a source that is not completely chaotic. The chaoticity and weight factor are evaluated as

measures of two- and three-pion correlations. The chaoticity always varies between 0 and 1, but the weight factor takes the value of a wide range and sometimes even diverges. The conventional weight factor includes the effects of the two-pion correlations in some models, yielding the anomalous behavior. We propose, in all model considered here, a new weight factor that has no such difficulty, but not expected to be valid universally. We find that the model of multi-coherent sources and one chaotic source could reproduce the chaoticity and the weight factor observed in the recent experiment.

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APPENDIX A: MULTI-COHERENT SOURCE MODEL

We assume that N coherent sources are created during a collision and that the position of the n -th source is X_n . The sources are uncorrelated with each other. A source current is defined as

$$J(x) = \sum_{n=1}^N j(x - X_n) e^{-i\theta_n}. \quad (\text{A1})$$

where θ_n is a unique random number varying from 0 to 2π . The average about θ_n is denoted by $\langle \cdots \rangle$. The Fourier transform of the source current is written as

$$J(p) = \sum_{n=1}^N j(p) e^{ip \cdot X_n - i\theta_n}, \quad (\text{A2})$$

where $j(p)$ is the Fourier transform of $j(x)$. The one-pion spectrum is

$$\begin{aligned} W_N(p_1) &= \frac{1}{(2\pi)^3 p_1^0} \langle |J(p_1)|^2 \rangle \\ &= \frac{|j(p_1)|^2}{(2\pi)^3 p_1^0} \sum_{n,m}^N e^{-ip_1 \cdot (X_n - X_m)} \langle e^{i\theta_n - i\theta_m} \rangle \\ &= a_1 N, \end{aligned} \quad (\text{A3})$$

where p^0 is on-shell and

$$a_1 = \frac{|j(p_1)|^2}{(2\pi)^3 p_1^0}. \quad (\text{A4})$$

The two-pion spectrum is

$$\begin{aligned} W_N(p_1, p_2) &= \frac{1}{(2\pi)^6 p_1^0 p_2^0} \langle |J(p_1)|^2 |J(p_2)|^2 \rangle \\ &= a_1 a_2 \left[N^2 + \sum_{n \neq m}^N e^{-i(p_1 - p_2) \cdot (X_m - X_n)} \right] \end{aligned} \quad (\text{A5})$$

The location of each small source, X_n , is assumed to obey a distribution $\rho(X_n)$, which is normalized to be unity, or $\int \rho(X_n) dX_n = 1$. Averaging the two-pion spectrum with the distribution, we obtain

$$\begin{aligned} \overline{W}_N(p_1, p_2) &= \int \rho(X_1) dX_1 \cdots \rho(X_N) dX_N W_N(p_1, p_2) \\ &= a_1 a_2 \left[N^2 + N(N-1) |\rho_{12}|^2 \right], \end{aligned} \quad (\text{A6})$$

where $\rho_{12} = \int \rho(x) dx e^{-i(p_1 - p_2) \cdot x}$. Note that $\sum_{i,j} A_{ij} = \sum_i A_{ii} + \sum_{i \neq j} A_{ij}$ and $\sum_{i \neq j} 1 = N(N-1)$.

The three-pion spectrum is

$$W_N(p_1, p_2, p_3) = \frac{1}{(2\pi)^6 p_1^0 p_2^0 p_3^0} \langle |J(p_1)|^2 |J(p_2)|^2 |J(p_3)|^2 \rangle, \quad (\text{A7})$$

and the averaged spectrum is evaluated as

$$\begin{aligned} \overline{W}_N(p_1, p_2, p_3) = & a_1 a_2 a_3 \left[N^3 + N^2(N-1) (|\rho_{12}|^2 + |\rho_{23}|^2 + |\rho_{31}|^2) \right. \\ & \left. + N(N-1)(N-2) \cdot 2\text{Re}(\rho_{12}\rho_{23}\rho_{31}) \right]. \end{aligned} \quad (\text{A8})$$

Note that $\sum_{i,j,k} A_{ijk} = \sum_i A_{iii} + \sum_{i \neq j} (A_{ijj} + A_{iji} + A_{jii}) + \sum_{i \neq j \neq k} A_{ijk}$ and that $\sum_{i \neq j \neq k} 1 = N(N-1)(N-2)$.

We next assume that the coherent sources obey the Poisson distribution. The Poisson distribution is

$$P_N^{(\text{usual})} = \frac{\alpha^N}{N!} e^{-\alpha} \quad \text{for } N = 0 \sim \infty. \quad (\text{A9})$$

The $N = 0$ event should be excluded because in such an event no pions are emitted. The distribution we use is renormalized as

$$P_N = \frac{\alpha^N}{N!} \frac{1}{e^\alpha - 1} \quad \text{for } N = 1 \sim \infty. \quad (\text{A10})$$

The expectation values change from those of the usual Poisson distribution,

$$\langle N \rangle_P = \frac{\alpha}{1 - e^{-\alpha}}, \quad (\text{A11})$$

$$\langle N(N-1) \rangle_P = \frac{\alpha^2}{1 - e^{-\alpha}}, \quad (\text{A12})$$

$$\langle N(N-1) \cdots (N-n) \rangle_P = \frac{\alpha^{n+1}}{1 - e^{-\alpha}}. \quad (\text{A13})$$

Pion spectra averaged with the Poisson distribution, $W = \sum_{N=1}^{\infty} P_N \overline{W}_N$, are

$$W(p_1) = a_1 \frac{\alpha}{1 - e^{-\alpha}}, \quad (\text{A14})$$

$$W(p_1, p_2) = \frac{a_1 a_2}{1 - e^{-\alpha}} \left[\alpha(\alpha + 1) + \alpha^2 |\rho_{12}|^2 \right], \quad (\text{A15})$$

$$\begin{aligned} W(p_1, p_2, p_3) = & \frac{a_1 a_2 a_3}{1 - e^{-\alpha}} \left[\alpha(\alpha^2 + 3\alpha + 1) \right. \\ & \left. + \alpha^2(\alpha + 2) (|\rho_{12}|^2 + |\rho_{23}|^2 + |\rho_{31}|^2) + \alpha^3 \cdot 2\text{Re}(\rho_{12}\rho_{23}\rho_{31}) \right]. \end{aligned} \quad (\text{A16})$$

The correlation functions are

$$C_2(p_1, p_2) = 1 + \frac{\alpha}{\alpha + 1} |\rho_{12}|^2, \quad (\text{A17})$$

$$\begin{aligned} C_3(p_1, p_2, p_3) = & 1 + \frac{\alpha(\alpha + 2)}{\alpha^2 + 3\alpha + 1} (|\rho_{12}|^2 + |\rho_{23}|^2 + |\rho_{31}|^2) \\ & + \frac{\alpha^2}{\alpha^2 + 3\alpha + 1} \cdot 2\text{Re}(\rho_{12}\rho_{23}\rho_{31}), \end{aligned} \quad (\text{A18})$$

where the correlation functions are normalized to be unity at $|p_i - p_j| \rightarrow \infty$. The chaoticity and weight factor are

$$\lambda = \frac{\alpha}{\alpha + 1}, \quad (\text{A19})$$

$$\omega = \frac{1}{2} \frac{2\alpha^2 + 2\alpha + 3}{\alpha^2 + 3\alpha + 1} \sqrt{\frac{\alpha + 1}{\alpha}}. \quad (\text{A20})$$

APPENDIX B: DERIVATION OF EQ. (50)

The two- and three-pion spectra can be written generally as

$$\begin{aligned} W_2(p_1, p_2) &= \langle a_1^\dagger a_1 \rangle \langle a_2^\dagger a_2 \rangle + \langle a_1^\dagger a_2 \rangle \langle a_2^\dagger a_1 \rangle + \langle\langle a_1^\dagger a_2^\dagger a_1 a_2 \rangle\rangle, \\ W_3(p_1, p_2, p_3) &= \langle a_1^\dagger a_1 \rangle \langle a_2^\dagger a_2 \rangle \langle a_3^\dagger a_3 \rangle + \sum_{(a,b,c)} \langle a_a^\dagger a_a \rangle \langle a_b^\dagger a_c \rangle \langle a_c^\dagger a_b \rangle + 2\text{Re} \left(\langle a_1^\dagger a_2 \rangle \langle a_2^\dagger a_3 \rangle \langle a_3^\dagger a_1 \rangle \right) \\ &\quad + \sum_{(a,b,c)} \langle a_a^\dagger a_a \rangle \langle\langle a_b^\dagger a_c^\dagger a_b a_c \rangle\rangle + 2\text{Re} \sum_{(a,b,c)} \langle a_a^\dagger a_b \rangle \langle\langle a_b^\dagger a_c^\dagger a_a a_c \rangle\rangle + \langle\langle a_1^\dagger a_2^\dagger a_3^\dagger a_1 a_2 a_3 \rangle\rangle, \end{aligned} \quad (\text{B1})$$

where a_i is an annihilation operator of momentum p_i . We assume that $\langle a_i \rangle = 0$, and therefore we don't consider the coherent source and partially coherent source in this appendix. $\langle\langle \dots \rangle\rangle$ corresponds to a cumulant, or a connected Green function. If we have a generating functional, $Z[z^*(p), z(p)]$, such as

$$\langle a_1^\dagger \dots a_n^\dagger a_1 \dots a_n \rangle = \frac{\delta^{2n} Z[z^*(p), z(p)]}{\delta z(p_1) \dots \delta z(p_n) \delta z^*(p_1) \dots \delta z(p_n)} \Big|_{z=0}, \quad (\text{B3})$$

then the cumulant is obtained as

$$\langle\langle a_1^\dagger \dots a_n^\dagger a_1 \dots a_n \rangle\rangle = \frac{\delta^{2n} (\ln Z[z^*(p), z(p)])}{\delta z(p_1) \dots \delta z(p_n) \delta z^*(p_1) \dots \delta z(p_n)} \Big|_{z=0}. \quad (\text{B4})$$

We introduce two assumptions. The first is

$$\langle\langle a_1^\dagger a_2^\dagger a_1 a_2 \rangle\rangle = A \langle a_1^\dagger a_1 \rangle \langle a_2^\dagger a_2 \rangle \quad (\text{B5})$$

$$\langle\langle a_1^\dagger a_2^\dagger a_3^\dagger a_1 a_2 a_3 \rangle\rangle = B \langle a_1^\dagger a_1 \rangle \langle a_2^\dagger a_2 \rangle \langle a_3^\dagger a_3 \rangle, \quad (\text{B6})$$

where A and B are constant. The second assumption is that the interference terms, such as $\langle a_1^\dagger a_2 \rangle \langle a_2^\dagger a_1 \rangle$, vanish by integrating over the momenta, for example,

$$\int d^3 p_1 d^3 p_2 \langle a_1^\dagger a_2 \rangle \langle a_2^\dagger a_1 \rangle = 0. \quad (\text{B7})$$

Thus we can obtain

$$\langle n(n-1) \rangle = (1+A) \langle n \rangle^2, \quad (\text{B8})$$

$$\langle n(n-1)(n-2) \rangle = (1+3A+B) \langle n \rangle^3. \quad (\text{B9})$$

Correlation functions at the zero relative momenta become

$$C_2(p, p) = 1 + \frac{1}{1 + A}, \quad (\text{B10})$$

$$C_3(p, p, p) = 1 + \frac{1}{1 + 3A + B} \{3 + 2 + 6A\}. \quad (\text{B11})$$

The terms 3, 2 and $6A$ between the braces in the above equation correspond to $\sum_{(a,b,c)} \langle a_a^\dagger a_a \rangle \langle a_b^\dagger a_c \rangle \langle a_c^\dagger a_b \rangle$, $2\text{Re} \left(\langle a_1^\dagger a_2 \rangle \langle a_2^\dagger a_3 \rangle \langle a_3^\dagger a_1 \rangle \right)$ and $2\text{Re} \sum_{(a,b,c)} \langle a_a^\dagger a_b \rangle \langle a_b^\dagger a_c^\dagger a_a a_c \rangle$ in Eq. (B2). To obtain the genuine three-pion correlation, we should subtract the first and third terms between the braces. Thus we define the new weight factor, considering the above discussion, as

$$\begin{aligned} \omega' &= \frac{1}{2} \frac{(1 + 3A + B) (C_3(p, p, p) - 1) - 3(1 + 2A) (C_2(p, p) - 1)}{[(1 + A) (C_2(p, p) - 1)]^{\frac{3}{2}}} \\ &= \frac{1}{2} \frac{\frac{\langle n(n-1)(n-2) \rangle}{\langle n \rangle^3} (C_3(p, p, p) - 1) - 3 \cdot \frac{2\langle n(n-1) \rangle - \langle n \rangle^2}{\langle n \rangle^2} (C_2(p, p) - 1)}{\left[\frac{\langle n(n-1) \rangle}{\langle n \rangle^2} (C_2(p, p) - 1) \right]^{\frac{3}{2}}} \end{aligned} \quad (\text{B12})$$

This equation corresponds to Eq. (50).

The two assumptions are valid for multi-coherent sources and one chaotic source with setting $A = \alpha^{-1}(1 - \epsilon)^2$ and $B = \alpha^{-2}(1 - \epsilon)^3$. For multi-coherent sources, the assumptions are not valid because of the factor $(1 - \exp(-\alpha))^{-1}$, in Eqs. (A14)-(A16), due to the renormalized Poisson distribution, Eq. (A10). In using the usual Poisson distribution instead of the renormalized one, the factor vanishes, and the assumptions become applicable with $A = \alpha^{-1}$ and $B = \alpha^{-2}$. The correlation functions are the same both in the cases of the usual and renormalized distributions. If we use the renormalized distributions and other assumptions, we can derive Eq. (B12), but the derivation is more complicated.

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FIGURES

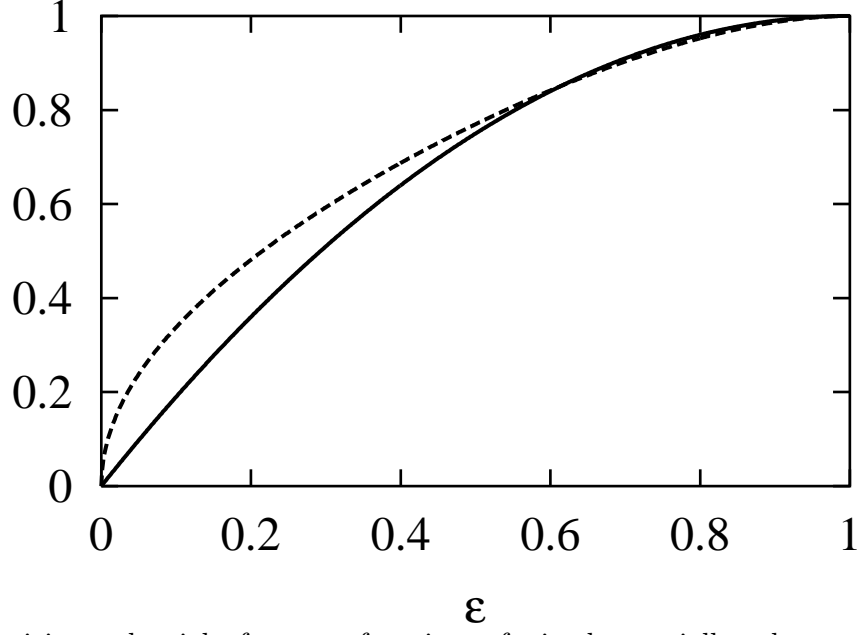


FIG. 1. Chaoticity and weight factor as functions of ϵ in the partially coherent model. Solid and dashed lines stand for the chaoticity and weight factor, respectively.

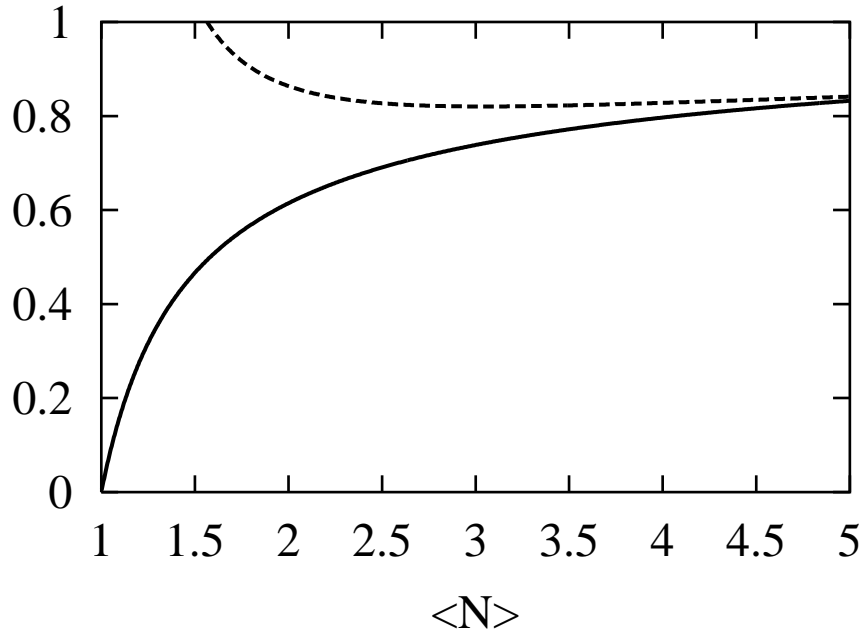


FIG. 2. Chaoticity and weight factor as functions of the mean number of coherent sources in the model of multi-coherent sources. Solid and dashed lines stand for the chaoticity and weight factor, respectively.

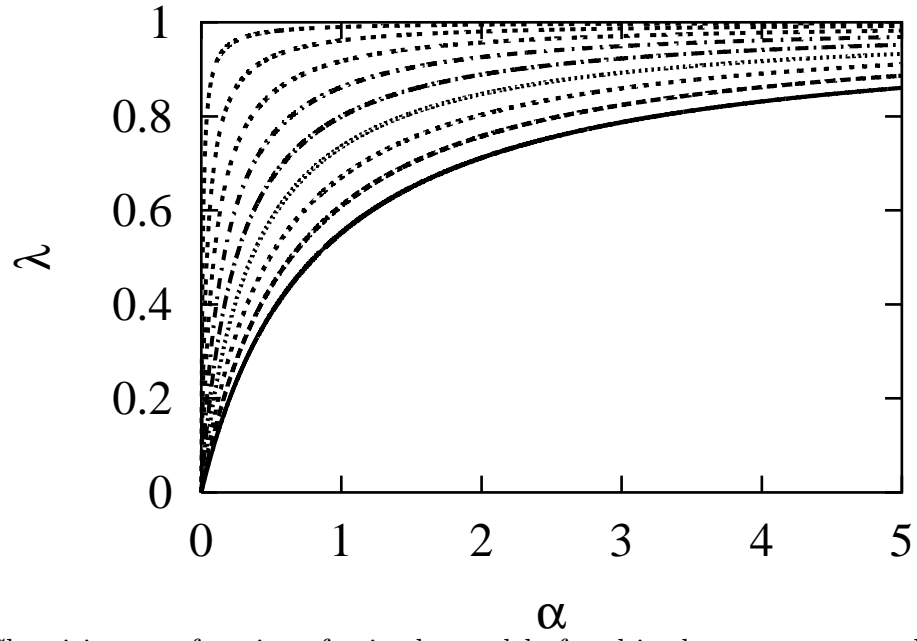


FIG. 3. Chaoticity as a function of α in the model of multi-coherent sources and one chaotic source. The lines from down to up correspond to ϵ varying from 0.1 to 0.9 with the step 0.1.

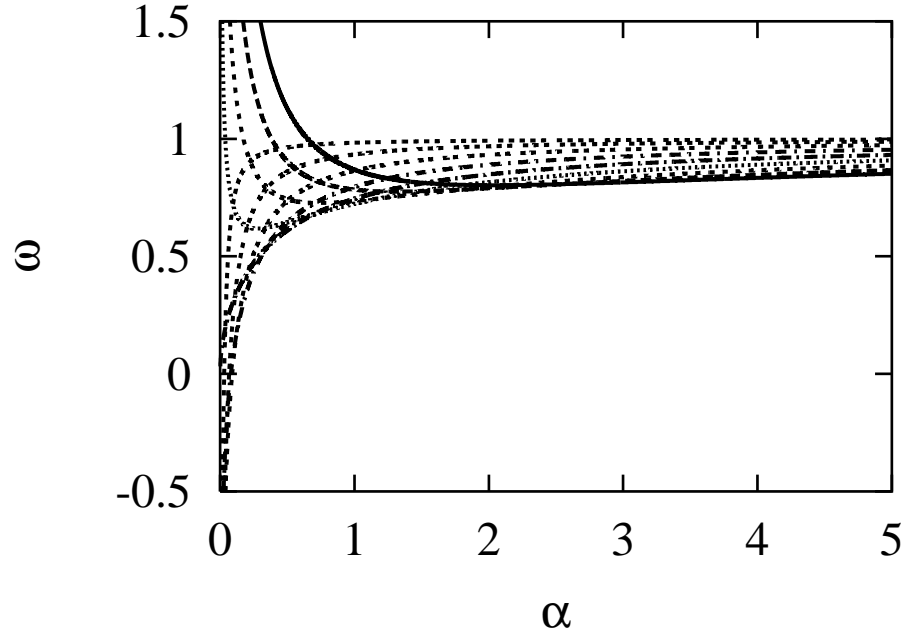


FIG. 4. Weight factor as a function of α . The model and lines are the same as in Fig. 3.

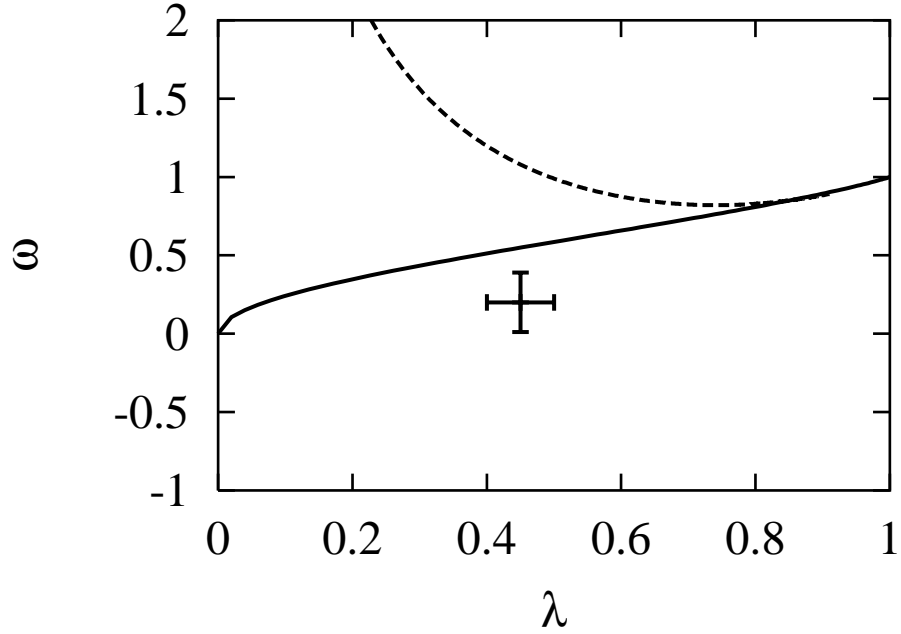


FIG. 5. Weight factors as functions of chaoticities. Solid and dashed lines stand for the partially coherent source and the multi-coherent sources, respectively. The plot with error-bars is the experimental data from NA44.

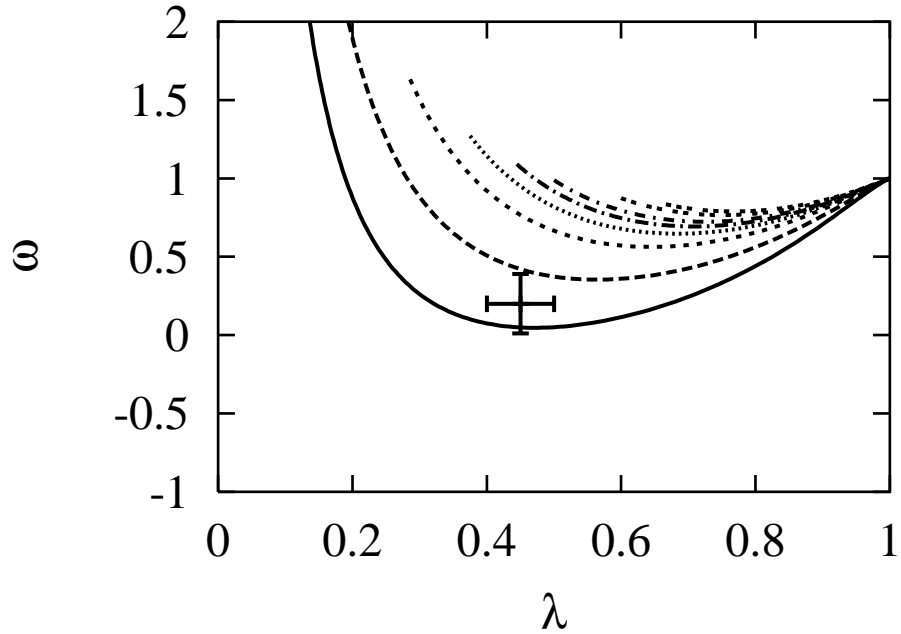


FIG. 6. Weight factors as functions of chaoticities in the model of multi-coherent sources and one chaotic source, varying ϵ from 0 to 1. The lines from down to up stand for $\alpha = 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0$, respectively. The plot with error-bars is the experimental data from NA44.